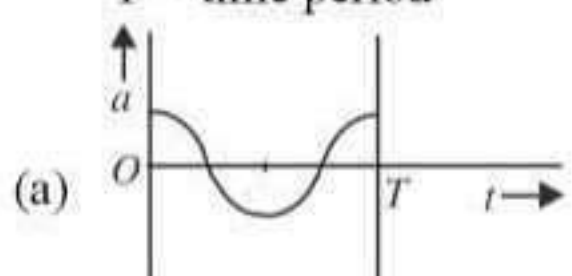
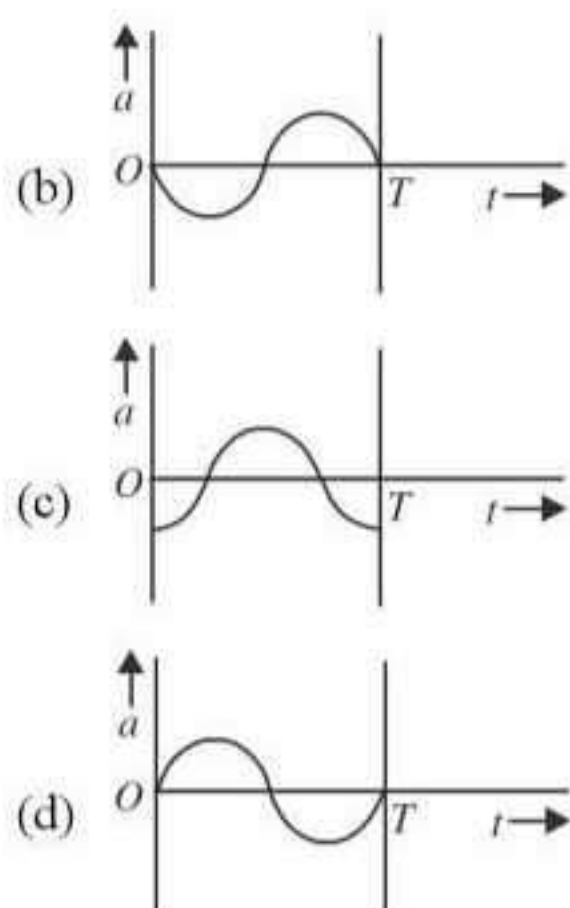


## Chapter 10. Oscillations

1. A spring of force constant  $k$  is cut into lengths of ratio 1 : 2 : 3. They are connected in series and the new force constant is  $k'$ . Then they are connected in parallel and force constant is  $k''$ . Then  $k' : k''$  is  
 (a) 1 : 9 (b) 1 : 11  
 (c) 1 : 14 (d) 1 : 6  
*(NEET 2017)*
2. A particle executes linear simple harmonic motion with an amplitude of 3 cm. When the particle is at 2 cm from the mean position, the magnitude of its velocity is equal to that of its acceleration. Then its time period in seconds is  
 (a)  $\frac{\sqrt{5}}{2\pi}$  (b)  $\frac{4\pi}{\sqrt{5}}$   
 (c)  $\frac{2\pi}{\sqrt{3}}$  (d)  $\frac{\sqrt{5}}{\pi}$   
*(NEET 2017)*
3. A body of mass  $m$  is attached to the lower end of a spring whose upper end is fixed. The spring has negligible mass. When the mass  $m$  is slightly pulled down and released, it oscillates with a time period of 3 s. When the mass  $m$  is increased by 1 kg, the time period of oscillations becomes 5 s. The value of  $m$  in kg is  
 (a)  $\frac{3}{4}$  (b)  $\frac{4}{3}$   
 (c)  $\frac{16}{9}$  (d)  $\frac{9}{16}$   
*(NEET-II 2016)*
4. A particle is executing a simple harmonic motion. Its maximum acceleration is  $\alpha$  and maximum velocity is  $\beta$ . Then, its time period of vibration will be  
 (a)  $\frac{\beta^2}{\alpha}$  (b)  $\frac{2\pi\beta}{\alpha}$   
 (c)  $\frac{\beta^2}{\alpha^2}$  (d)  $\frac{\alpha}{\beta}$  *(2015)*
5. A particle is executing SHM along a straight line. Its velocities at distances  $x_1$  and  $x_2$  from the mean position are  $V_1$  and  $V_2$ , respectively. Its time period is  
 (a)  $2\pi\sqrt{\frac{V_1^2 + V_2^2}{x_1^2 + x_2^2}}$  (b)  $2\pi\sqrt{\frac{V_1^2 - V_2^2}{x_1^2 - x_2^2}}$   
 (c)  $2\pi\sqrt{\frac{x_1^2 + x_2^2}{V_1^2 + V_2^2}}$  (d)  $2\pi\sqrt{\frac{x_2^2 - x_1^2}{V_1^2 - V_2^2}}$   
*(2015 Cancelled)*
6. When two displacements represented by  $y_1 = a \sin(\omega t)$  and  $y_2 = b \cos(\omega t)$  are superimposed the motion is  
 (a) simple harmonic with amplitude  $\sqrt{a^2 + b^2}$   
 (b) simple harmonic with amplitude  $\frac{(a+b)}{2}$   
 (c) not a simple harmonic  
 (d) simple harmonic with amplitude  $\frac{a}{b}$   
*(2015 Cancelled)*
7. The oscillation of a body on a smooth horizontal surface is represented by the equation,  
 $X = A \cos(\omega t)$   
 where  $X$  = displacement at time  $t$   
 $\omega$  = frequency of oscillation  
 Which one of the following graphs shows correctly the variation  $a$  with  $t$ ?  
 Here  $a$  = acceleration at time  $t$   
 $T$  = time period
- (a) 



(2014)

8. A particle of mass  $m$  oscillates along  $x$ -axis according to equation  $x = a \sin \omega t$ . The nature of the graph between momentum and displacement of the particle is

(a) Circle  
 (b) Hyperbola  
 (c) Ellipse  
 (d) Straight line passing through origin

(Karnataka NEET 2013)

9. Out of the following functions representing motion of a particle which represents SHM

(1)  $y = \sin \omega t - \cos \omega t$

(2)  $y = \sin^3 \omega t$

(3)  $y = 5 \cos \left( \frac{3\pi}{4} - 3\omega t \right)$

(4)  $y = 1 + \omega t + \omega^2 t^2$

(a) Only (1)  
 (b) Only (4) does not represent SHM  
 (c) Only (1) and (3)  
 (d) Only (1) and (2)

(2011)

10. Two particles are oscillating along two close parallel straight lines side by side, with the same frequency and amplitudes. They pass each other, moving in opposite directions when their displacement is half of the amplitude. The mean positions of the two particles lie on a straight line perpendicular to the paths of the two particles. The phase difference is

(a)  $\frac{\pi}{6}$  (b) 0

(c)  $\frac{2\pi}{3}$  (d)  $\pi$  (Mains 2011)

11. The displacement of a particle along the  $x$ -axis is given by  $x = a \sin^2 \omega t$ . The motion of the particle corresponds to

(a) simple harmonic motion of frequency  $\omega/\pi$   
 (b) simple harmonic motion of frequency  $3\omega/2\pi$   
 (c) non simple harmonic motion  
 (d) simple harmonic motion of frequency  $\omega/2\pi$

(2010)

12. The period of oscillation of a mass  $M$  suspended from a spring of negligible mass is  $T$ . If along with it another mass  $M$  is also suspended, the period of oscillation will now be

(a)  $T$  (b)  $\frac{T}{\sqrt{2}}$

(c)  $2T$  (d)  $\sqrt{2}T$

(2010)

13. A simple pendulum performs simple harmonic motion about  $x = 0$  with an amplitude  $a$  and time period  $T$ . The speed of the pendulum at  $x = a/2$  will be

(a)  $\frac{\pi a}{T}$  (b)  $\frac{3\pi^2 a}{T}$

(c)  $\frac{\pi a \sqrt{3}}{T}$  (d)  $\frac{\pi a \sqrt{3}}{2T}$

(2009)

14. Which one of the following equations of motion represents simple harmonic motion?

(a) Acceleration  $= -k(x + a)$

(b) Acceleration  $= k(x + a)$

(c) Acceleration  $= kx$

(d) Acceleration  $= -k_0 x + k_1 x^2$

where  $k, k_0, k_1$  and  $a$  are all positive.

(2009)

15. Two simple harmonic motions of angular frequency 100 and 1000  $\text{rad s}^{-1}$  have the same displacement amplitude. The ratio of their maximum acceleration is

(a) 1 : 10<sup>3</sup> (b) 1 : 10<sup>4</sup>

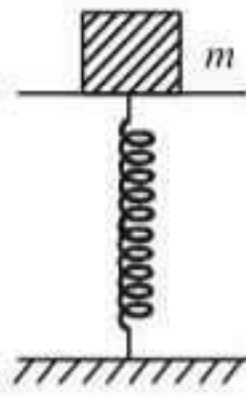
(c) 1 : 10 (d) 1 : 10<sup>2</sup> (2008)

16. A particle executes simple harmonic oscillation with an amplitude  $a$ . The period of oscillation is  $T$ . The minimum time taken by the particle to travel half of the amplitude from the equilibrium position is

(a)  $T/8$  (b)  $T/12$

(c)  $T/2$  (d)  $T/4$  (2007)

17. A mass of 2.0 kg is put on a flat pan attached to a vertical spring fixed on the ground as shown in the figure. The mass of the spring and the pan is negligible.



When pressed slightly and released the mass executes a simple harmonic motion. The spring constant is 200 N/m. What should be the minimum amplitude of the motion so that the mass gets detached from the pan (take  $g = 10 \text{ m/s}^2$ ).

- (a) 10.0 cm  
 (b) any value less than 12.0 cm  
 (c) 4.0 cm  
 (d) 8.0 cm. (2007)

18. The particle executing simple harmonic motion has a kinetic energy  $K_0 \cos^2 \omega t$ . The maximum values of the potential energy and the total energy are respectively

- (a)  $K_0/2$  and  $K_0$  (b)  $K_0$  and  $2K_0$   
 (c)  $K_0$  and  $K_0$  (d) 0 and  $2K_0$

(2007)

19. The phase difference between the instantaneous velocity and acceleration of a particle executing simple harmonic motion is

- (a)  $\pi$  (b)  $0.707\pi$   
 (c) zero (d)  $0.5\pi$ . (2007)

20. A rectangular block of mass  $m$  and area of cross-section  $A$  floats in a liquid of density  $\rho$ . If it is given a small vertical displacement from equilibrium it undergoes with a time period  $T$ , then

- (a)  $T \propto \frac{1}{\sqrt{m}}$  (b)  $T \propto \sqrt{\rho}$   
 (c)  $T \propto \frac{1}{\sqrt{A}}$  (d)  $T \propto \frac{1}{\rho}$  (2006)

21. The circular motion of a particle with constant speed is

- (a) periodic but not simple harmonic  
 (b) simple harmonic but not periodic  
 (c) period and simple harmonic  
 (d) neither periodic not simple harmonic.

(2005)

22. A particle executing simple harmonic motion of amplitude 5 cm has maximum speed of 31.4 cm/s. The frequency of its oscillation is

- (a) 4 Hz (b) 3 Hz  
 (c) 2 Hz (d) 1 Hz. (2004)

23. Two springs of spring constants  $k_1$  and  $k_2$  are joined in series. The effective spring constant of the combination is given by

- (a)  $\sqrt{k_1 k_2}$  (b)  $(k_1 + k_2)/2$   
 (c)  $k_1 + k_2$  (d)  $k_1 k_2 / (k_1 + k_2)$   
 (2004)

24. Which one of the following statements is true for the speed  $v$  and the acceleration  $a$  of a particle executing simple harmonic motion?

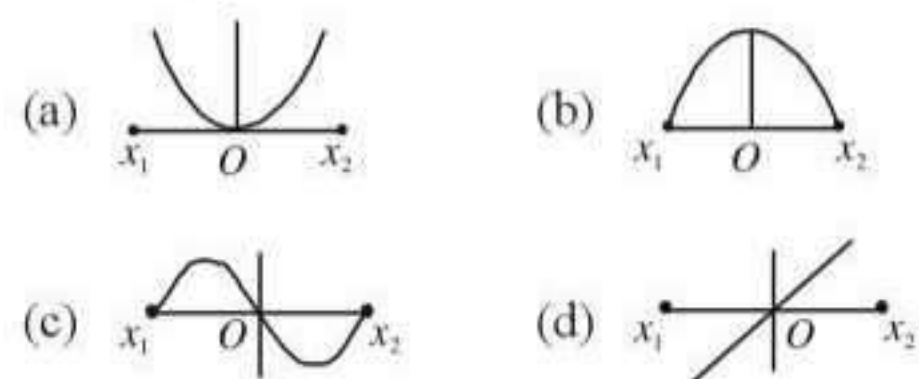
- (a) When  $v$  is maximum,  $a$  is maximum.  
 (b) Value of  $a$  is zero, whatever may be the value of  $v$ .  
 (c) When  $v$  is zero,  $a$  is zero.  
 (d) When  $v$  is maximum,  $a$  is zero. (2003)

25. The potential energy of a simple harmonic oscillator when the particle is half way to its end point is

- (a)  $\frac{2}{3}E$  (b)  $\frac{1}{8}E$   
 (c)  $\frac{1}{4}E$  (d)  $\frac{1}{2}E$  (2003)

where  $E$  is the total energy.

26. A particle of mass  $m$  oscillates with simple harmonic motion between points  $x_1$  and  $x_2$ , the equilibrium position being  $O$ . Its potential energy is plotted. It will be as given below in the graph



(2003)

27. The time period of mass suspended from a spring is  $T$ . If the spring is cut into four equal parts and the same mass is suspended from one of the parts, then the new time period will be

- (a)  $T/4$  (b)  $T$   
 (c)  $T/2$  (d)  $2T$  (2003)

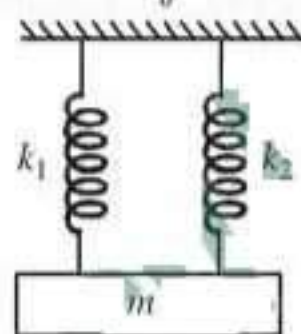
28. In case of a forced vibration, the resonance peak becomes very sharp when the
- damping force is small
  - restoring force is small
  - applied periodic force is small
  - quality factor is small
- (2003)

29. Displacement between maximum potential energy position and maximum kinetic energy position for a particle executing simple harmonic motion is
- $\pm a/2$
  - $+a$
  - $\pm a$
  - $-1$
- (2002)

30. When an oscillator completes 100 oscillations its amplitude reduced to  $\frac{1}{3}$  of initial value. What will be its amplitude, when it completes 200 oscillations?

- $\frac{1}{8}$
  - $\frac{2}{3}$
  - $\frac{1}{6}$
  - $\frac{1}{9}$
- (2002)

31. A mass is suspended separately by two different springs in successive order then time periods is  $t_1$  and  $t_2$  respectively. If it is connected by both spring as shown in figure then time period is  $t_0$ , the correct relation is



- $t_0^2 = t_1^2 + t_2^2$
  - $t_0^{-2} = t_1^{-2} + t_2^{-2}$
  - $t_0^{-1} = t_1^{-1} + t_2^{-1}$
  - $t_0 = t_1 + t_2$
- (2002)

32. The total energy of particle performing SHM depend on
- $k, a, m$
  - $k, a$
  - $k, a, x$
  - $k, x$
- (2001)

33. Two masses  $M_A$  and  $M_B$  are hung from two strings of length  $l_A$  and  $l_B$  respectively. They are executing SHM with frequency relation  $f_A = 2f_B$ , then relation

- $l_A = \frac{l_B}{4}$ , does not depend on mass
- $l_A = 4l_B$ , does not depend on mass

(c)  $l_A = 2l_B$  and  $M_A = 2M_B$

(d)  $l_A = \frac{l_B}{2}$  and  $M_A = \frac{M_B}{2}$ . (2000)

34. The bob of simple pendulum having length  $l$ , is displaced from mean position to an angular position  $\theta$  with respect to vertical. If it is released, then velocity of bob at equilibrium position

- $\sqrt{2gl(1 - \cos\theta)}$
- $\sqrt{2gl(1 + \cos\theta)}$
- $\sqrt{2gl \cos\theta}$
- $\sqrt{2gl}$

(2000)

35. Time period of a simple pendulum is 2 sec. If its length is increased by 4 times, then its period becomes

- 8 sec
  - 12 sec
  - 16 sec
  - 4 sec
- (1999)

36. A particle, with restoring force proportional to displacement and resisting force proportional to velocity is subjected to a force  $F \sin \omega t$ . If the amplitude of the particle is maximum for  $\omega = \omega_1$  and the energy of the particle maximum for  $\omega = \omega_2$ , then

- $\omega_1 \neq \omega_0$  and  $\omega_2 = \omega_0$
  - $\omega_1 = \omega_0$  and  $\omega_2 = \omega_0$
  - $\omega_1 = \omega_0$  and  $\omega_2 \neq \omega_0$
  - $\omega_1 \neq \omega_0$  and  $\omega_2 \neq \omega_0$
- (1989, 1998)

37. Two simple pendulums of length 5 m and 20 m respectively are given small linear displacement in one direction at the same time. They will again be in the phase when the pendulum of shorter length has completed \_\_\_\_\_ oscillations.

- 2
  - 1
  - 5
  - 3
- (1998)

38. A mass  $m$  is vertically suspended from a spring of negligible mass; the system oscillates with a frequency  $n$ . What will be the frequency of the system, if a mass  $4m$  is suspended from the same spring?

- $\frac{n}{2}$
  - $4n$
  - $\frac{n}{4}$
  - $2n$
- (1998)

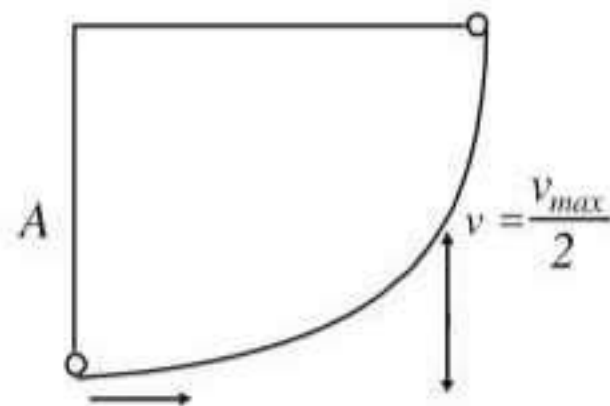
39. If the length of a simple pendulum is increased by 2%, then the time period

- increases by 1%
  - decreases by 1%
  - increases by 2%
  - decreases by 2%
- (1997)

40. Two SHM's with same amplitude and time period, when acting together in perpendicular directions with a phase difference of  $\pi/2$ , give rise to  
 (a) straight motion (b) elliptical motion  
 (c) circular motion (d) none of these.  
 (1997)

41. A particle starts with S.H.M. from the mean position as shown in the figure. Its amplitude is  $A$  and its time period is  $T$ . At one time, its speed is half that of the maximum speed. What is this displacement?

- (a)  $\frac{2A}{\sqrt{3}}$   
 (b)  $\frac{3A}{\sqrt{2}}$   
 (c)  $\frac{\sqrt{2}A}{3}$   
 (d)  $\frac{\sqrt{3}A}{2}$

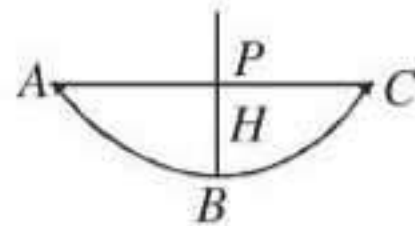


(1996)

42. A linear harmonic oscillator of force constant  $2 \times 10^6$  N/m and amplitude 0.01 m has a total mechanical energy of 160 J. Its  
 (a) P.E. is 160 J (b) P.E. is zero  
 (c) P.E. is 100 J (d) P.E. is 120 J  
 (1996)

43. A simple pendulum with a bob of mass  $m$  oscillates from  $A$  to  $C$  and back to  $A$  such that  $PB$  is  $H$ . If the acceleration due to gravity is  $g$ , then the velocity of the bob as it passes through  $B$  is

- (a)  $mgH$   
 (b)  $\sqrt{2gH}$   
 (c) zero  
 (d)  $2gH$ .



(1995)

44. In a simple harmonic motion, when the displacement is one-half the amplitude, what fraction of the total energy is kinetic?  
 (a)  $1/2$  (b)  $3/4$   
 (c) zero (d)  $1/4$ . (1995)

45. A body of mass 5 kg hangs from a spring and oscillates with a time period of  $2\pi$  seconds. If the ball is removed, the length of the spring will decrease by

- (a)  $g/k$  metres (b)  $k/g$  metres  
 (c)  $2\pi$  metres (d)  $g$  metres. (1994)

46. A particle executes S.H.M. along  $x$ -axis. The force acting on it is given by  
 (a)  $A \cos(kx)$  (b)  $Ae^{-kx}$   
 (c)  $Akx$  (d)  $-Akx$ .  
 (1994, 1988)

47. A seconds pendulum is mounted in a rocket. Its period of oscillation will decrease when rocket is  
 (a) moving down with uniform acceleration  
 (b) moving around the earth in geostationary orbit  
 (c) moving up with uniform velocity  
 (d) moving up with uniform acceleration.  
 (1994)

48. A loaded vertical spring executes S.H.M. with a time period of 4 sec. The difference between the kinetic energy and potential energy of this system varies with a period of  
 (a) 2 sec (b) 1 sec  
 (c) 8 sec (d) 4 sec. (1994)

49. A body executes simple harmonic motion with an amplitude  $A$ . At what displacement from the mean position is the potential energy of the body is one fourth of its total energy?  
 (a)  $A/4$  (b)  $A/2$   
 (c)  $3A/4$   
 (d) Some other fraction of  $A$  (1993)

50. A simple harmonic oscillator has an amplitude  $A$  and time period  $T$ . The time required by it to travel from  $X = A$  to  $A = A/2$  is  
 (a)  $T/6$  (b)  $T/4$   
 (c)  $T/3$  (d)  $T/2$  (1992)

51. If a simple harmonic oscillator has got a displacement of 0.02 m and acceleration equal to  $0.02$  m/s<sup>2</sup> at any time, the angular frequency of the oscillator is equal to  
 (a) 10 rad/s (b) 0.1 rad/s  
 (c) 100 rad/s (d) 1 rad/s (1992)

52. A simple pendulum is suspended from the roof of a trolley which moves in a horizontal direction with an acceleration  $a$ , then the time period is given by  $T = 2\pi\sqrt{l/g}$ , where  $g$  is equal to  
 (a)  $g$  (b)  $g - a$   
 (c)  $g + a$  (d)  $\sqrt{g^2 + a^2}$   
 (1991)

53. A body is executing simple harmonic motion. When the displacements from the mean position is 4 cm and 5 cm, the corresponding velocities of the body is 10 cm/sec and 8 cm/sec. Then the time period of the body is  
 (a)  $2\pi$  sec (b)  $\pi/2$  sec  
 (c)  $\pi$  sec (d)  $3\pi/2$  sec  
 (1991)
54. The angular velocity and the amplitude of a simple pendulum is  $\omega$  and  $a$  respectively. At a displacement  $x$  from the mean position if its kinetic energy is  $T$  and potential energy is  $V$ , then the ratio of  $T$  to  $V$  is  
 (a)  $\frac{(a^2 - x^2\omega^2)}{x^2\omega^2}$  (b)  $\frac{x^2\omega^2}{(a^2 - x^2\omega^2)}$   
 (c)  $\frac{(a^2 - x^2)}{x^2}$  (d)  $\frac{x^2}{(a^2 - x^2)}$   
 (1991)
55. The composition of two simple harmonic motions of equal periods at right angle to each other and with a phase difference of  $\pi$  results in the displacement of the particle along  
 (a) circle  
 (b) figure of eight  
 (c) straight line  
 (d) ellipse  
 (1990)
56. A mass  $m$  is suspended from the two coupled springs connected in series. The force constant for springs are  $k_1$  and  $k_2$ . The time period of the suspended mass will be  
 (a)  $T = 2\pi\sqrt{\frac{m}{k_1 - k_2}}$   
 (b)  $T = 2\pi\sqrt{\frac{mk_1k_2}{k_1 + k_2}}$   
 (c)  $T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$   
 (d)  $T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1k_2}}$   
 (1990)

## Answer Key

1.	(b)	2.	(b)	3.	(d)	4.	(b)	5.	(d)	6.	(a)	7.	(c)	8.	(c)	9.	(c)	10.	(c)
11.	(c)	12.	(d)	13.	(c)	14.	(*)	15.	(d)	16.	(b)	17.	(a)	18.	(c)	19.	(d)	20.	(c)
21.	(a)	22.	(d)	23.	(d)	24.	(d)	25.	(c)	26.	(a)	27.	(c)	28.	(a)	29.	(c)	30.	(d)
31.	(b)	32.	(b)	33.	(a)	34.	(a)	35.	(d)	36.	(b)	37.	(a)	38.	(a)	39.	(a)	40.	(c)
41.	(d)	42.	(c)	43.	(b)	44.	(b)	45.	(d)	46.	(d)	47.	(d)	48.	(a)	49.	(b)	50.	(a)
51.	(a)	52.	(d)	53.	(c)	54.	(c)	55.	(c)	56.	(d)								



## EXPLANATIONS

1. (b) : Let us assume, the length of spring be  $l$ . When we cut the spring into ratio of length 1 : 2 : 3, we get three springs of lengths  $\frac{l}{6}$ ,  $\frac{2l}{6}$  and  $\frac{3l}{6}$  with force constant,

$$\therefore k_1 = \frac{kl}{l_1} = \frac{kl}{l/6} = 6k$$

$$k_2 = \frac{kl}{l_2} = \frac{kl}{2l/6} = 3k$$

$$k_3 = \frac{kl}{l_3} = \frac{kl}{3l/6} = 2k$$

When connected in series,

$$\frac{1}{k'} = \frac{1}{6k} + \frac{1}{3k} + \frac{1}{2k} = \frac{1+2+3}{6k} = \frac{1}{k}$$

$$\therefore k' = k$$

When connected in parallel,

$$k'' = 6k + 3k + 2k = 11k$$

$$\frac{k'}{k''} = \frac{k}{11k} = \frac{1}{11}$$

2. (b) : Given,  $A = 3$  cm,  $x = 2$  cm

The velocity of a particle in simple harmonic motion is given as

$$v = \omega \sqrt{A^2 - x^2}$$

and magnitude of its acceleration is

$$a = \omega^2 x$$

Given  $|v| = |a|$

$$\therefore \omega \sqrt{A^2 - x^2} = \omega^2 x$$

$$\omega x = \sqrt{A^2 - x^2} \quad \text{or} \quad \omega^2 x^2 = A^2 - x^2$$

$$\omega^2 = \frac{A^2 - x^2}{x^2} = \frac{9 - 4}{4} = \frac{5}{4}$$

$$\omega = \frac{\sqrt{5}}{2}$$

$$\text{Time period, } T = \frac{2\pi}{\omega} = 2\pi \cdot \frac{2}{\sqrt{5}} = \frac{4\pi}{\sqrt{5}} \text{ s}$$

3. (d) : Time period of spring - block system,

$$T = 2\pi \sqrt{\frac{m}{k}}$$

For given spring,  $T \propto \sqrt{m}$

$$\frac{T_1}{T_2} = \sqrt{\frac{m_1}{m_2}}$$

Here,  $T_1 = 3$  s,  $m_1 = m$ ,  $T_2 = 5$  s,  $m_2 = m + 1$ ,  $m = ?$

$$\frac{3}{5} = \sqrt{\frac{m}{m+1}} \quad \text{or} \quad \frac{9}{25} = \frac{m}{m+1}$$

$$25m = 9m + 9 \Rightarrow 16m = 9$$

$$\therefore m = \frac{9}{16} \text{ kg}$$

4. (b) : If  $A$  and  $\omega$  be amplitude and angular frequency of vibration, then

$$\alpha = \omega^2 A \quad \dots (i)$$

$$\text{and } \beta = \omega A \quad \dots (ii)$$

Dividing eqn. (i) by eqn. (ii), we get

$$\frac{\alpha}{\beta} = \frac{\omega^2 A}{\omega A} = \omega$$

$\therefore$  Time period of vibration is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{(\alpha/\beta)} = \frac{2\pi\beta}{\alpha}$$

5. (d) : In SHM, velocities of a particle at distances  $x_1$  and  $x_2$  from mean position are given by

$$V_1^2 = \omega^2 (a^2 - x_1^2) \quad \dots (i)$$

$$V_2^2 = \omega^2 (a^2 - x_2^2) \quad \dots (ii)$$

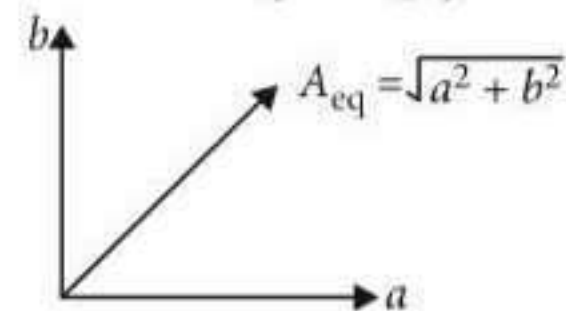
From equations (i) and (ii), we get

$$V_1^2 - V_2^2 = \omega^2 (x_2^2 - x_1^2)$$

$$\omega = \sqrt{\frac{V_1^2 - V_2^2}{x_2^2 - x_1^2}} \quad \therefore T = 2\pi \sqrt{\frac{x_2^2 - x_1^2}{V_1^2 - V_2^2}}$$

6. (a) : Here,  $y_1 = a \sin \omega t$

$$y_2 = b \cos \omega t = b \sin \left( \omega t + \frac{\pi}{2} \right)$$



Hence, resultant motion is SHM with amplitude

$$\sqrt{a^2 + b^2}.$$

7. (c) : Here,  $X = A \cos \omega t$

$$\therefore \text{Velocity, } v = \frac{dX}{dt} = \frac{d}{dt} (A \cos \omega t)$$

$$= -A\omega \sin \omega t$$

$$\text{Acceleration, } a = \frac{dv}{dt} = \frac{d}{dt}(-A\omega \sin \omega t) \\ = -A\omega^2 \cos \omega t$$

Hence the variation of  $a$  with  $t$  is correctly shown by graph (c).

$$8. \text{ (c) : } x = a \sin \omega t \text{ or } \frac{x}{a} = \sin \omega t \quad \dots (i)$$

$$\text{Velocity, } v = \frac{dx}{dt} = a\omega \cos \omega t$$

$$\frac{v}{a\omega} = \cos \omega t \quad \dots (ii)$$

Squaring and adding (i) and (ii), we get

$$\frac{x^2}{a^2} + \frac{v^2}{a^2\omega^2} = \sin^2 \omega t + \cos^2 \omega t$$

$$\frac{x^2}{a^2} + \frac{v^2}{a^2\omega^2} = 1$$

It is an equation of ellipse.

Hence, the graph between velocity and displacement is an ellipse.

Momentum of the particle =  $mv$

$\therefore$  The nature of graph of the momentum and displacement is same as that of velocity and displacement.

$$9. \text{ (c) : } y = \sin \omega t - \cos \omega t \\ = \sqrt{2} \left[ \frac{1}{\sqrt{2}} \sin \omega t - \frac{1}{\sqrt{2}} \cos \omega t \right] = \sqrt{2} \sin \left( \omega t - \frac{\pi}{4} \right)$$

It represents a SHM with time period  $T = \frac{2\pi}{\omega}$ .

$$y = \sin^3 \omega t = \frac{1}{4} [3 \sin \omega t - \sin 3\omega t]$$

It represents a periodic motion with time period

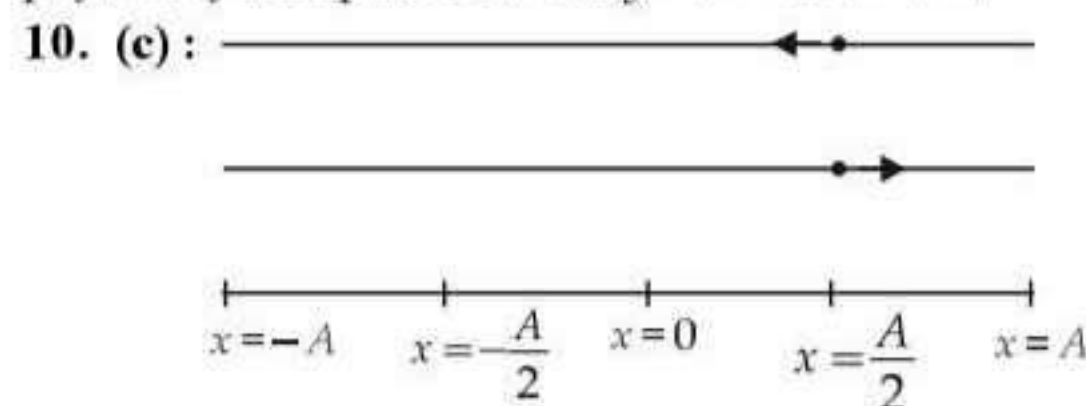
$$T = \frac{2\pi}{\omega} \text{ but not SHM.}$$

$$y = 5 \cos \left( \frac{3\pi}{4} - 3\omega t \right) \\ = 5 \cos \left( 3\omega t - \frac{3\pi}{4} \right) \quad [\because \cos(-\theta) = \cos \theta]$$

It represents a SHM with time period,  $T = \frac{2\pi}{3\omega}$ .

$$y = 1 + \omega t + \omega^2 t^2$$

It represents a non-periodic motion. Also it is not physically acceptable as the  $y \rightarrow \infty$  as  $t \rightarrow \infty$ .



The time taken by the particle to travel from

$$x = 0 \text{ to } x = \frac{A}{2} \text{ is } \frac{T}{12}.$$

The time taken by the particle to travel from

$$x = A \text{ to } x = \frac{A}{2} \text{ is } \frac{T}{6}.$$

$$\text{Time difference} = \frac{T}{6} + \frac{T}{6} = \frac{T}{3}$$

$$\text{Phase difference, } \phi = \frac{2\pi}{T} \times \text{Time difference}$$

$$= \frac{2\pi}{T} \times \frac{T}{3} = \frac{2\pi}{3}$$

$$11. \text{ (c) : } x = a \sin^2 \omega t$$

$$= a \left( \frac{1 - \cos 2\omega t}{2} \right) \quad (\because \cos 2\theta = 1 - 2\sin^2 \theta)$$

$$= \frac{a}{2} - \frac{a \cos 2\omega t}{2}$$

$$\therefore \text{Velocity, } v = \frac{dx}{dt} = \frac{2\omega a \sin 2\omega t}{2} = \omega a \sin 2\omega t$$

$$\text{Acceleration, } a = \frac{dv}{dt} = 2\omega^2 a \cos 2\omega t$$

For the given displacement  $x = a \sin^2 \omega t$ ,

$$a \propto -x \text{ is not satisfied.}$$

Hence, the motion of the particle is non simple harmonic motion.

**Note :** The given motion is a periodic motion with a time period

$$T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$$

12. (d) : A mass  $M$  is suspended from a massless spring of spring constant  $k$  as shown in figure (a). Then,

Time period of oscillation is

$$T = 2\pi \sqrt{\frac{M}{k}} \quad \dots (i)$$

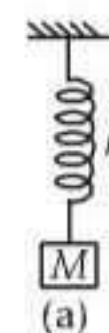
When another mass  $M$  is also suspended with it as shown in figure (b). Then,

Time period of oscillation is

$$T' = 2\pi \sqrt{\frac{M+M}{k}} = 2\pi \sqrt{\frac{2M}{k}} \\ = \sqrt{2} \left( 2\pi \sqrt{\frac{M}{k}} \right) = \sqrt{2} T \quad (\text{Using (i)})$$

13. (c) : For simple harmonic motion,

$$v = \omega \sqrt{a^2 - x^2}. \text{ When } x = \frac{a}{2},$$





$$\text{When } x = \frac{a}{2}, v = \omega \sqrt{a^2 - \frac{a^2}{4}} = \omega \sqrt{\frac{3}{4}a^2}.$$

$$\text{As } \omega = \frac{2\pi}{T}, \therefore v = \frac{2\pi}{T} \cdot \frac{\sqrt{3}}{2}a \Rightarrow v = \frac{\pi\sqrt{3}a}{T}.$$

14. (\*) : Simple harmonic motion is defined as follows

$$\text{Acceleration } \frac{d^2y}{dt^2} = -\omega^2x$$

The negative sign is very important in simple harmonic motion. Acceleration is independent of any initial displacement of equilibrium position.

Then acceleration =  $-\omega^2x$ .

\* Option not given.

15. (d) :  $\omega_1 = 100 \text{ rad s}^{-1}$ ;  $\omega_2 = 1000 \text{ rad s}^{-1}$ .

Maximum acceleration of (1) =  $-\omega_1^2A$

Maximum acceleration of (2) =  $-\omega_2^2A$

$$\therefore \frac{\text{accln (1)}}{\text{accln (2)}} = \frac{\omega_1^2}{\omega_2^2} = \frac{(100)^2}{(1000)^2} = \frac{1}{100}$$

$$a(1) : a(2) = 1 : 100.$$

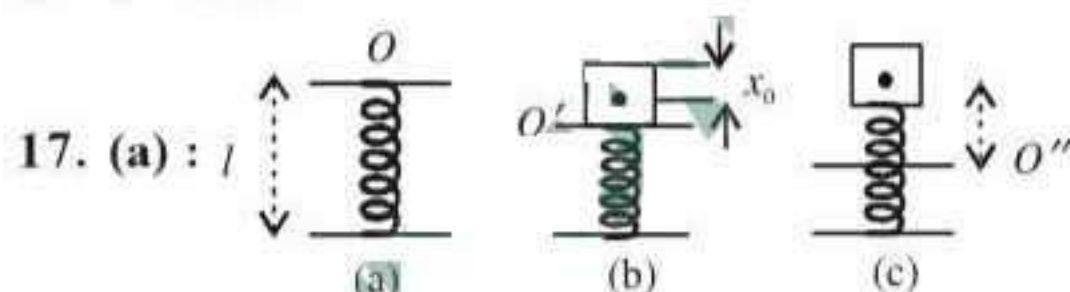
16. (b) :  $x(t) = a \sin \omega t$  (from the equilibrium position)

At  $x(t) = a/2$

$$\therefore \frac{a}{2} = a \sin(\omega t)$$

$$\Rightarrow \sin\left(\frac{\pi}{6}\right) = \sin(\omega t) \quad \text{or, } \frac{\pi}{6} = \frac{2\pi t}{T} \quad \left[ \because \omega = \frac{2\pi}{T} \right]$$

or  $t = T/12$ .



17. (a) : The spring has a length  $l$ . When  $m$  is placed over it, the equilibrium position becomes  $O'$ .

If it is pressed from  $O'$  (the equilibrium position) to  $O''$ ,  $O'O''$  is the amplitude.

$$OO' = \frac{mg}{k} = \frac{2 \times 10}{200} = 0.10 \text{ m.}$$

$$mg = kx_0.$$

If the restoring force  $mA\omega^2 > mg$ , then the mass will move up with acceleration, detached from the pan.

$$\text{i.e. } A > \frac{g}{k/m} \Rightarrow A > \frac{20}{200} > 0.10 \text{ m.}$$

The amplitude  $> 10 \text{ cm}$ .

i.e. the minimum is just greater than  $10 \text{ cm}$ .

(The actual compression will include  $x_0$  also. But when talking of amplitude, it is always from the

equilibrium position with respect to which the mass is oscillating.

18. (c) : Kinetic energy + potential energy = total energy

When kinetic energy is maximum, potential energy is zero and vice versa.

$\therefore$  Maximum potential energy = total energy.

$$0 + K_0 = K_0$$

(K.E. + P.E. = total energy).

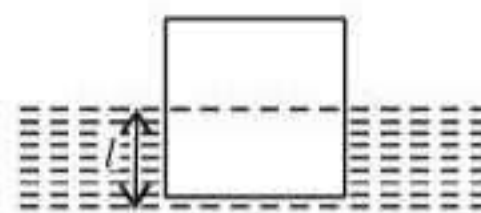
19. (d) : Let  $y = A \sin \omega t$

$$\frac{dy}{dt} = A\omega \cos \omega t = A\omega \sin\left(\omega t + \frac{\pi}{2}\right)$$

Acceleration =  $-A\omega^2 \sin \omega t$

The phase difference between acceleration and velocity is  $\pi/2$ .

20. (c) : Let  $l$  be the length of block immersed in liquid as shown in the figure. When the block is floating,



$$\therefore mg = A l \rho m$$

If the block is given vertical displacement  $y$  then the effective restoring force is

$$F = -[A(l+y)\rho g - mg] = -[A(l+y)\rho g - A l \rho g] = -A \rho g y$$

Restoring force =  $-[A \rho g]y$ . As this  $F$  is directed towards its equilibrium position of block, so if the block is left free, it will execute simple harmonic motion.

Here inertia factor = mass of block =  $m$

Spring factor =  $A \rho g$

$$\therefore \text{Time period} = T = 2\pi \sqrt{\frac{m}{A \rho g}}$$

$$\text{i.e. } T \propto \frac{1}{\sqrt{A}}$$

21. (a)

22. (d) :  $a = 5 \text{ cm}$ ,  $v_{\text{max}} = 31.4 \text{ cm/s}$

$$v_{\text{max}} = \omega a \Rightarrow 31.4 = 2\pi \nu \times 5$$

$$\Rightarrow 31.4 = 10 \times 3.14 \times \nu \Rightarrow \nu = 1 \text{ Hz.}$$

23. (d) : When the spring joined in series the total extension in spring is

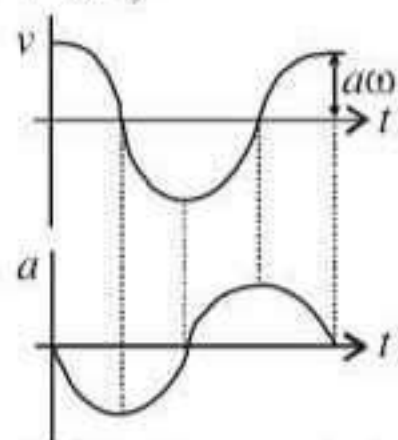
$$\Rightarrow y = y_1 + y_2 = \frac{-F}{k_1} - \frac{F}{k_2}$$

$$\Rightarrow y = -F \left[ \frac{1}{k_1} + \frac{1}{k_2} \right]$$

Thus spring constant in this case becomes

$$\Rightarrow k = \frac{k_1 k_2}{k_1 + k_2}$$

**24. (d) :** In simple harmonic motion velocity  
 $= A\omega \sin(\omega t + \pi/2)$



acceleration  $= A\omega^2 \sin(\omega t + \pi)$  from this we can easily find out that when  $v$  is maximum, then  $a$  is zero.

**25. (c) :** Potential energy of simple harmonic oscillator  $= \frac{1}{2} m\omega^2 y^2$

for  $y = \frac{a}{2}$ , P.E.  $= \frac{1}{2} m\omega^2 \frac{a^2}{4}$

$$\Rightarrow \text{P.E.} = \frac{1}{4} \left( \frac{1}{2} m\omega^2 a^2 \right) = \frac{E}{4}$$

**26. (a) :** Potential energy of particle performing SHM varies parabolically in such a way that at mean position it becomes zero and maximum at extreme position.

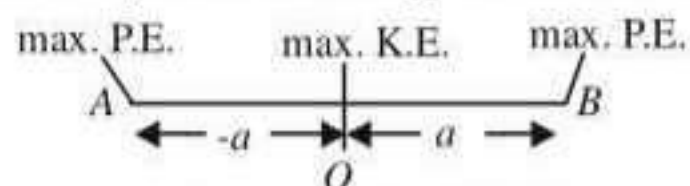
**27. (c) :** Let  $k$  be the force constant of spring. If  $k'$  is the force constant of each part, then

$$\frac{1}{k} = \frac{4}{k'} \Rightarrow k' = 4k$$

$$\therefore \text{Time period} = 2\pi \sqrt{\frac{m}{4k}} = \frac{1}{2} \times 2\pi \sqrt{\frac{m}{k}} = \frac{T}{2}$$

**28. (a) :** Smaller damping gives a taller and narrower resonance peak.

**29. (c) :** For a simple harmonic motion between  $A$  and  $B$ , with  $O$  as the mean position, maximum kinetic energy of the particle executing SHM will be at  $O$  and maximum potential energy will be at  $A$  and  $B$ .



$a$  is the amplitude of SHM

$\therefore$  Displacement between maximum potential energy and maximum kinetic energy is  $\pm a$ .

**30. (d) :** This is a case of damped vibration as the amplitude of vibration is decreasing with time. Amplitude of vibrations at any instant  $t$  is given by  $a = a_0 e^{-bt}$ , where  $a_0$  is the initial amplitude of

vibrations and  $b$  is the damping constant.

Now, when  $t = 100T$ ,  $a = a_0/3$  [ $T$  is time period]

Let the amplitude be  $a'$  at  $t = 200T$ .

i.e. after completing 200 oscillations.

$$\therefore a = a_0/3 = a_0 e^{-100Tb} \quad \dots(i)$$

$$\text{and } a' = a_0 e^{-200Tb} \quad \dots(ii)$$

$$\text{From (i), } \frac{1}{3} = e^{-100Tb} \quad \therefore e^{-200Tb} = 1/9$$

$$\text{From (ii), } a' = a_0 \times \frac{1}{9} = \frac{a_0}{9}$$

$\therefore$  The amplitude will be reduced to 1/9 of initial value.

**31. (b) :** The time period of a spring mass system as shown in figure 1 is given by  $T = 2\pi \sqrt{m/k}$ , where  $k$  is the spring constant.

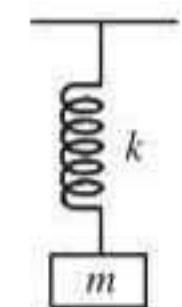


Figure 1

$$\therefore t_1 = 2\pi \sqrt{m/k_1} \quad \dots(i)$$

$$\text{and } t_2 = 2\pi \sqrt{m/k_2} \quad \dots(ii)$$

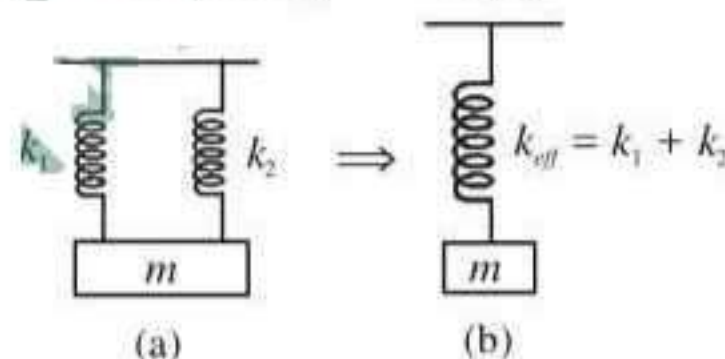


figure 2

Now, when they are connected in parallel as shown in figure 2(a), the system can be replaced by a single spring of spring constant,  $k_{eff} = k_1 + k_2$ .

$$[\text{Since } mg = k_1 x + k_2 x = k_{eff} x]$$

$$\therefore t_0 = 2\pi \sqrt{m/k_{eff}} = 2\pi \sqrt{m/(k_1 + k_2)} \quad \dots(iii)$$

$$\text{From (i), } \frac{1}{t_1^2} = \frac{1}{4\pi^2} \times \frac{k_1}{m} \quad \dots(iv)$$

$$\text{From (ii), } \frac{1}{t_2^2} = \frac{1}{4\pi^2} \times \frac{k_2}{m} \quad \dots(v)$$

$$\text{From (iii), } \frac{1}{t_0^2} = \frac{1}{4\pi^2} \times \frac{k_1 + k_2}{m} \quad \dots(vi)$$

$$(iv) + (v) = \frac{1}{t_1^2} + \frac{1}{t_2^2} = \frac{1}{4\pi^2 m} (k_1 + k_2) = \frac{1}{t_0^2}$$

$$\therefore t_0^{-2} = t_1^{-2} + t_2^{-2}$$

$$\mathbf{32. (b) : \text{Energy} = \frac{1}{2} m\omega^2 a^2 = \frac{1}{2} ka^2}$$

$$\mathbf{33. (a) : f_A = 2f_B}$$

$$\Rightarrow \frac{1}{2\pi} \sqrt{\frac{g}{l_A}} = 2 \times \frac{1}{2\pi} \sqrt{\frac{g}{l_B}} \quad \text{or, } \frac{1}{l_A} = 4 \times \frac{1}{l_B}$$

$$\text{or, } l_A = \frac{l_B}{4}, \text{ which does not depend on mass.}$$

34. (a) : In  $\Delta OAC$ ,  $\cos\theta = A/l$

or,  $OA = l \cos\theta$

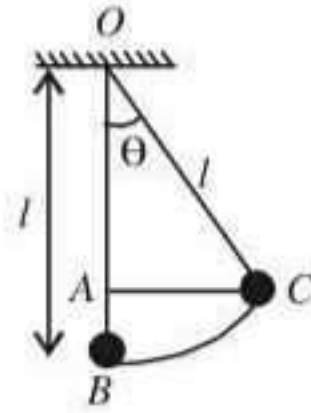
$\therefore AB = l(1 - \cos\theta) = h$

At point, C the velocity of bob = 0.

The vertical acceleration =  $g$

$\therefore v^2 = 2gh$

or,  $v = \sqrt{2gl(1 - \cos\theta)}$ .



35. (d) : Time period of a simple pendulum is given

by  $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{l}$ .

$\therefore \frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$  or,  $T_2 = 2T_1 = 4$  sec.

36. (b) : The amplitude and velocity resonance occurs at the same frequency.

At resonance, i.e.,  $\omega_1 = \omega_0$  and  $\omega_2 = \omega_0$  the amplitude and energy of the particle would be maximum.

37. (a) : Frequency of the pendulum  $\nu_{l=5} = \frac{1}{2\pi}\sqrt{\frac{g}{5}}$

$$\nu_{l=20} = \frac{1}{2\pi}\sqrt{\frac{g}{20}}$$

$\therefore \frac{\nu_{l=5}}{\nu_{l=20}} = \sqrt{\frac{20}{5}} = 2 \Rightarrow \nu_{l=5} = 2\nu_{l=20}$

As shorter length pendulum has frequency double the larger length pendulum. Therefore shorter pendulum should complete 2 oscillations before they will be again in phase.

38. (a) :  $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ ;  $n' = \frac{1}{2\pi}\sqrt{\frac{k}{4m}}$

$\therefore n' = n/2$

39. (a) :  $l_2 = 1.02l_1$ ; Time period  $(T) = 2\pi\sqrt{\frac{l}{g}} \propto \sqrt{l}$

Therefore  $\frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}} = \sqrt{\frac{1.02l_1}{l_1}} = 1.01$ .

Thus time period increased by 1%.

40. (c) :  $x = a \sin\omega t$

$$y = a \sin(\omega t + \pi/2) = a \cos\omega t$$

or,  $\frac{x}{y} = \frac{\sin\omega t}{\cos\omega t} = \tan\omega t$  or,  $\frac{x}{y} = \frac{x}{\sqrt{a^2 - x^2}}$ ,

or,  $y^2 = a^2 - x^2$  or,  $x^2 + y^2 = a^2$ .

It is an equation of a circle.

41. (d) : Maximum velocity,  $v_{\max} = A\omega$

According to question,  $\frac{v_{\max}}{2} = \frac{A\omega}{2} = \omega\sqrt{A^2 - y^2}$

$$\frac{A^2}{4} = A^2 - y^2 \Rightarrow y^2 = A^2 - \frac{A^2}{4} \Rightarrow y = \frac{\sqrt{3}A}{2}$$

42. (c) : Force constant ( $k$ ) =  $2 \times 10^6$  N/m; Amplitude ( $x$ ) = 0.01 m and total mechanical energy = 160 J.

$$\text{Potential energy} = \frac{1}{2}kx^2 = \frac{1}{2} \times (2 \times 10^6) \times (0.01)^2 = 100 \text{ J.}$$

43. (b) : Potential energy at A (or C) = Kinetic energy at B. Thus  $\frac{1}{2}mv_B^2 = mgH$  or  $v_B = \sqrt{2gH}$ .

44. (b) : Displacement ( $x$ ) =  $\frac{a}{2}$ .

Total energy =  $\frac{1}{2}m\omega^2a^2$  and kinetic energy when displacement is ( $x$ )

$$= \frac{1}{2}m\omega^2(a^2 - x^2)$$

$$= \frac{1}{2}m\omega^2\left(a^2 - \left(\frac{a}{2}\right)^2\right) = \frac{3}{4}\left(\frac{1}{2}m\omega^2a^2\right)$$

Therefore fraction of the total energy at

$$x = \frac{\frac{3}{4}\left(\frac{1}{2}m\omega^2a^2\right)}{\frac{1}{2}m\omega^2a^2} = \frac{3}{4}$$

45. (d) : Mass ( $m$ ) = 5 kg and time period ( $T$ ) =  $2\pi$  sec.

Therefore time period  $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow \sqrt{\frac{5}{k}} = 1$

or  $k = 5$  N/m. According to Hooke's Law,  $F = -kl$ .

Therefore decrease in length ( $l$ ) =  $-\frac{F}{k} = -\frac{5g}{5} = -g$  metres

46. (d) : For simple harmonic motion,  $\frac{d^2x}{dt^2} \propto -x$ .

Therefore force acting on the particle =  $-A kx$ .

47. (d) : Period of oscillation  $T = 2\pi\sqrt{\frac{l}{g}}$ . Therefore

$T$  will decrease when acceleration ( $g$ ) increases. And  $g$  will increase when the rocket moves up with a uniform acceleration.

48. (a) : Time period = 4 sec. In one simple harmonic oscillation, the same kinetic and potential energies are repeated two times. So the difference will be 2 seconds.

49. (b) : PE =  $\frac{1}{2}M\omega^2x^2 = \frac{1}{4}E = \frac{1}{4}\left(\frac{1}{2}M\omega^2A^2\right)$

where total energy  $E = \frac{1}{2}M\omega^2A^2 \therefore x = \frac{A}{2}$

50. (a) : For S.H.M.,  $x = A \sin\left(\frac{2\pi}{T}t\right)$

when  $x = A$ ,  $A = A \sin\left(\frac{2\pi}{T}t\right)$

$\therefore \sin\left(\frac{2\pi}{T}t\right) = 1 \Rightarrow \sin\left(\frac{2\pi}{T}t\right) = \sin\left(\frac{\pi}{2}\right)$

$\Rightarrow t = (T/4)$

When  $x = \frac{A}{2}$ ,  $\frac{A}{2} = A \sin\left(\frac{2\pi}{T}t\right)$

or  $\sin\frac{\pi}{6} = \sin\left(\frac{2\pi}{T}t\right)$  or  $t = (T/12)$

Now, time taken to travel from  $x = A$  to  $x = A/2 = T/4 - T/12 = T/6$

51. (a) : Acceleration =  $-\omega^2$  displacement

$$\omega^2 = \frac{\text{acceleration}}{\text{displacement}} = \frac{2.0}{0.02}$$

$$\omega^2 = 100 \text{ or } \omega = 10 \text{ rad/s}$$

52. (d) : The effective value of acceleration due to gravity is  $\sqrt{(a^2 + g^2)}$

53. (c) : For simple harmonic motion velocity

$$v = \omega\sqrt{a^2 - x^2} \text{ at displacement } x.$$

$$10 = \omega\sqrt{a^2 - 16} \quad \text{---(i)}$$

$$8 = \omega\sqrt{a^2 - 25} \quad \text{---(ii)}$$

$$\frac{100}{\omega^2} = a^2 - 16 \quad \text{---(iii)}$$

$$\frac{64}{\omega^2} = a^2 - 25 \quad \text{---(iv)}$$

$\therefore$  Equation (iii) - (iv) gives  $\frac{36}{\omega^2} = 9$

$\Rightarrow \omega = 2 \text{ rad/s}$

or  $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi \text{ sec}$

54. (c) : P.E,  $V = \frac{1}{2}m\omega^2x^2$

and KE,  $T = \frac{1}{2}m\omega^2(a^2 - x^2)$

$\therefore \frac{T}{V} = \frac{a^2 - x^2}{x^2}$

55. (c) :  $x = a \sin \omega t$

and  $y = b \sin(\omega t + \pi) = -b \sin \omega t.$

or  $\frac{x}{a} = -\frac{y}{b}$  or  $y = -\frac{b}{a}x$

It is an equation of a straight line

56. (d) : The effective spring constant of two springs

in series is  $k = \frac{k_1 k_2}{k_1 + k_2}$

Time period,  $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$

